

SEMESTER-IV

COURSE 3: RING THEORY

Theory

Credits: 4

5 hrs/week

Course Outcomes

After successful completion of this course, the student will be able to

1. acquire the basic knowledge of rings, fields and integral domains
2. get the knowledge of subrings and ideals
3. construct composition tables for finite quotient rings
4. study the homomorphisms and isomorphisms with applications.
5. get the idea of division algorithm of polynomials over a field.

Course Content

Unit – 1

Rings and Fields

Definition of a ring and Examples – Basic properties – Boolean rings - Fields – Divisors of 0 and Cancellation Laws – Integral Domains – Division ring - The Characteristic of a Ring, Integral domain and Field – NonCommutative Rings - Matrices over a field – The Quaternion ring.

Unit – 2

Subrings and Ideals

Definition and examples of Subrings – Necessary and sufficient conditions for a subset to be a subring – Algebra of Subrings – Centre of a ring – left, right and two sided ideals – Algebra of ideals – Equivalence of a field and a commutative ring without proper ideals

Unit III: Principal ideals and Quotient rings

Definition of a Principal ideal ring(Domain) – Every field is a PID – The ring of integers is a PID – Example of a ring which is not a PIR – Cosets – Algebra of cosets – Quotient rings – Construction of composition tables for finite quotient rings of the ring \mathbb{Z} of integers and the ring \mathbb{Z}_n of integers modulo n .

Unit – 4

Homomorphism of Rings

Homomorphism of Rings – Definition and Elementary properties – Kernel of a homomorphism – Isomorphism – Fundamental theorems of homomorphism of rings – Maximal and prime Ideals – Prime Fields

Unit – 5

Rings of Polynomials

Polynomials in an indeterminate – The Evaluation morphism -- The Division Algorithm in $F[x]$ – Irreducible Polynomials – Ideal Structure in $F[x]$ – Uniqueness of Factorization $F[x]$.

Activities

Seminar/ Quiz/ Assignments/ Applications of ring theory concepts to Real life Problem /Problem Solving Sessions.

Text book

Modern Algebra by A.R.Vasishta and A.K.Vasishta, Krishna Prakashan Media Pvt. Ltd.

Reference books

1. A First Course in Abstract Algebra by John. B. Farleigh, Narosa Publishing House.
2. Linear Algebra by Stephen. H. Friedberg and Others, Pearson Education India

SEMESTER-IV

COURSE 4: INTRODUCTION TO REAL ANALYSIS

Theory

Credits: 4

5 hrs/week

Course Outcomes

After successful completion of this course, the student will be able to

1. get clear idea about the real numbers and real valued functions.
2. obtain the skills of analysing the concepts and applying appropriate methods for testing convergence of a sequence/ series.
3. test the continuity and differentiability and Riemann integration of a function.
4. know the geometrical interpretation of mean value theorems.
5. know about the fundamental theorem of integral calculus

Course Contents

Unit – 1

REAL NUMBERS, REAL SEQUENCES

The algebraic and order properties of \mathbb{R} - Absolute value and Real line - Completeness property of \mathbb{R} - Applications of supremum property - intervals. (**No question is to be set from this portion**) Sequences and their limits - Range and Boundedness of Sequences - Limit of a sequence and Convergent sequence - The Cauchy's criterion - properly divergent sequences - Monotone sequences - Necessary and Sufficient condition for Convergence of Monotone Sequence - Limit Point of Sequence - Subsequences and the Bolzano-weierstrass theorem – Cauchy Sequences – Cauchy's general principle of convergence.

Unit – 2

INFINITE SERIES

Introduction to series - convergence of series - Cauchy's general principle of convergence for series tests for convergence of series - Series of non-negative terms - P-test - Cauchy's n^{th} root test - D'Alembert's Test - Alternating Series - Leibnitz Test.

Unit – 3

LIMIT & CONTINUITY

Real valued Functions - Boundedness of a function - Limits of functions - Some extensions of the limit concept - Infinite Limits - Limits at infinity (**No question is to be set from this portion**). Continuous functions - Combinations of continuous functions - Continuous Functions on intervals - uniform continuity.

Unit – 4

DIFFERENTIATION AND MEAN VALUE THEOREMS

The derivability of a function at a point and on an interval - Derivability and continuity of a function - Mean value Theorems - Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

Unit – 5

RIEMANN INTEGRATION

Riemann Integral - Riemann integral functions - Darboux theorem - Necessary and sufficient condition for \mathbb{R} integrability - Properties of integrable functions - Fundamental theorem of integral calculus - integral as the limit of a sum - Mean value Theorems.

Activities

Seminar/ Quiz/ Assignments/ Applications of Real Analysis to Real life Problem /Problem Solving Sessions.

TextBook

An Introduction to Real Analysis by Robert G. Bartle and Donald R. Sherbert, John Wiley and sons Pvt. Ltd

ReferenceBooks

1. Elements of Real Analysis by Shanthi Narayan and Dr. M. D. Raisinghania, S. Chand & Company Pvt. Ltd., New Delhi.
2. Principles of Mathematical Analysis by Walter Rudin, McGraw-Hill Ltd.
